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## Low Risk Investment in the German Stock Market

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### Summary:

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DAX index values hold information on the near future development of the market in the time range of some weeks, but not in the range of six months or even longer. The information contained in the price data can be used to estimate trends and thus enable low risk and high yield engagements in the stock market.

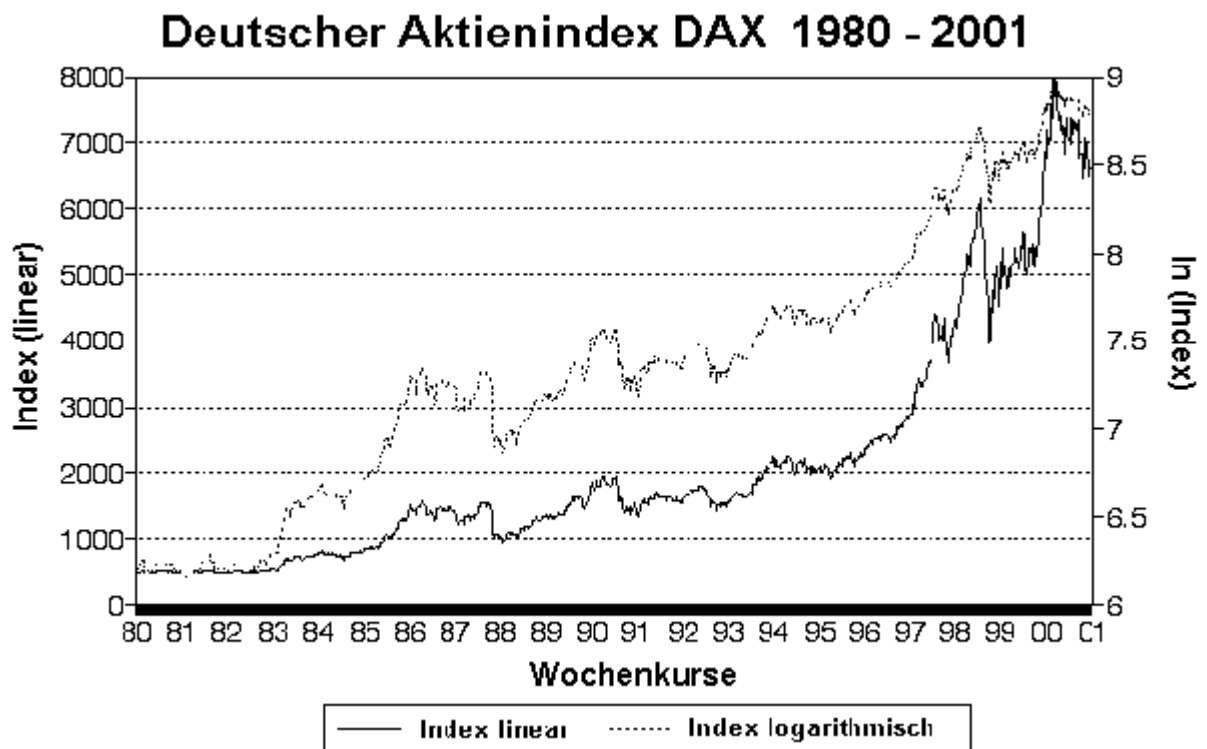
The information content of price data was estimated by comparing them to price surrogate data, and quantified using established methods from information science (chi<sup>2</sup>-test of independence, information entropy test, mutual information). Suitable weights for estimating robust trends were found using genetic algorithms.

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The long phase of serious losses in the international stock markets nourish the doubts about stocks as a safe home for investment capital. The markets now seems to have stabilized at a low level, but what can we expect from the future? Will the uptrend resume, like after the crash in 1987 or will we share the fate of Japanese investors, who have experience a decline for the last twelve years. I don't know. And I am convinced that nobody knows. Increasing evidence suggests that stock markets are chaotic systems. One of the central traits of chaotic dynamic systems is that they are not predictable in the long run. In this case the method of 'buy and hold' believing in the eternal uptrend would not be a responsible investment strategy. Responsible, that means, low risk investment in the stock market is possible. But to achieve that goal we would have to adjust our investments to our actual knowledge about the market and not merely let us be guided by our hopes and wishes about the future market developments.

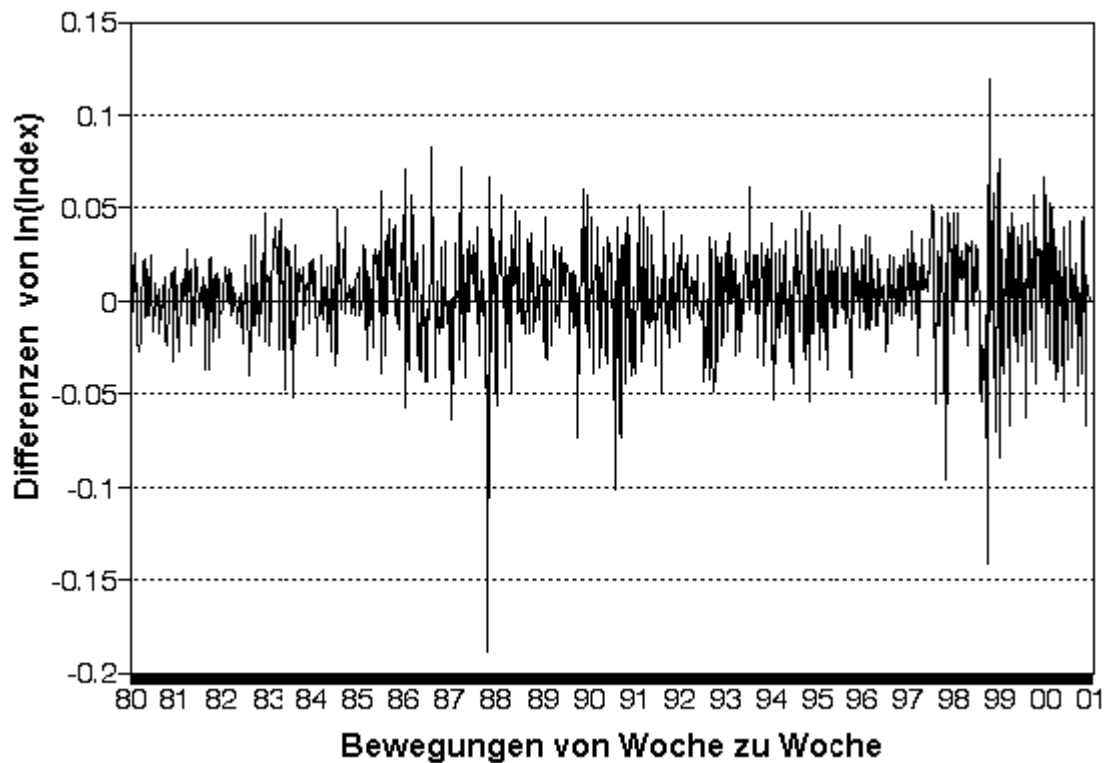
'Efficient market' theory assumes that the market price at any time reflects all available information about its present state and the expected development. Only new, unexpected events should therefore influence the price. Since future events occur at random, price forecasts should be impossible.

Thanks to information science and the mathematical theory of non linear statistics we do not depend on assumptions when we have to decide if and how much information is contained in data. Non linear statistics had been in the shadow for a long time until chaos researchers discovered that they are indispensable for making the decision if data come from a system that is either complex, which means ordered, but non periodic, or if they are a sequence of random events. With the aid of established methods like the  $\chi^2$ -Test of independence, the test for information entropy and the mutual information test we can find out, if data contain information and how much information it is. And we can even show the dynamics of the information decline with time passing. In this paper we will test the null hypothesis, that is DAX data do not contain information on the future development of the stock market using the tests mentioned above and by comparing the real market data with five surrogate time series.



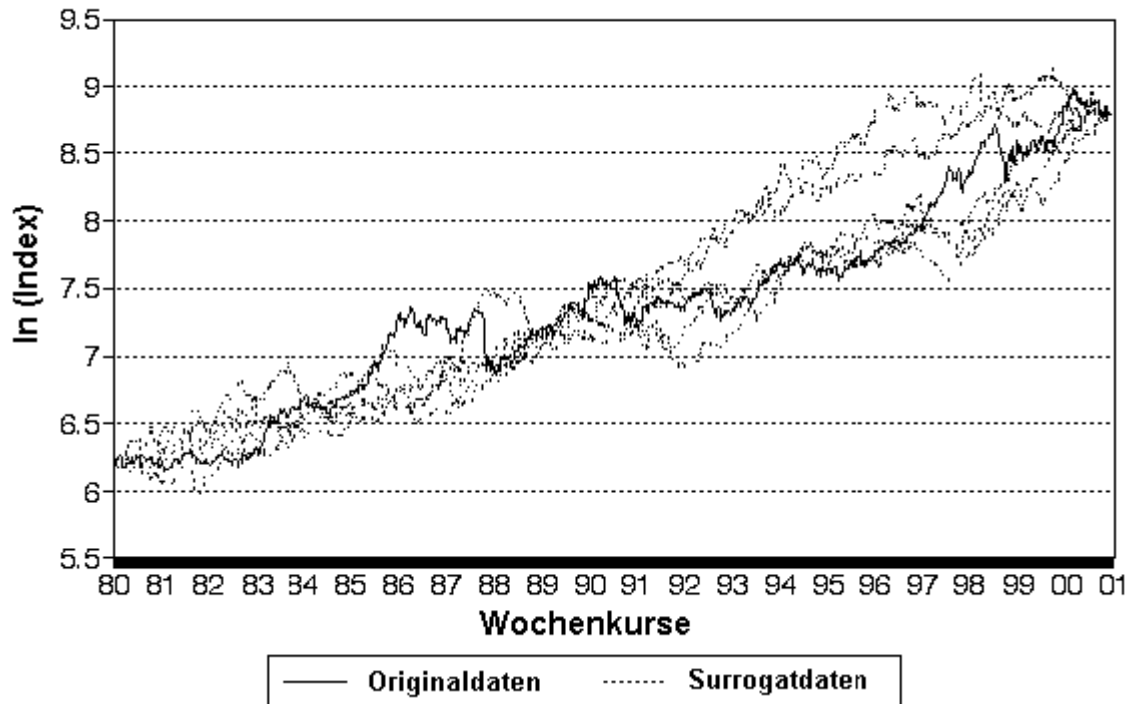
The first figure shows the DAX time series in linear and in logarithmic scale. We can see that using the logarithmic scale the long term trend can be approximated by a straight line. How would the DAX series look without a trend? A simple method to detrend data is to use the ratio of successive data (the difference of logarithms) and plotting them versus time.

## Deutscher Aktienindex DAX 1980 - 2001



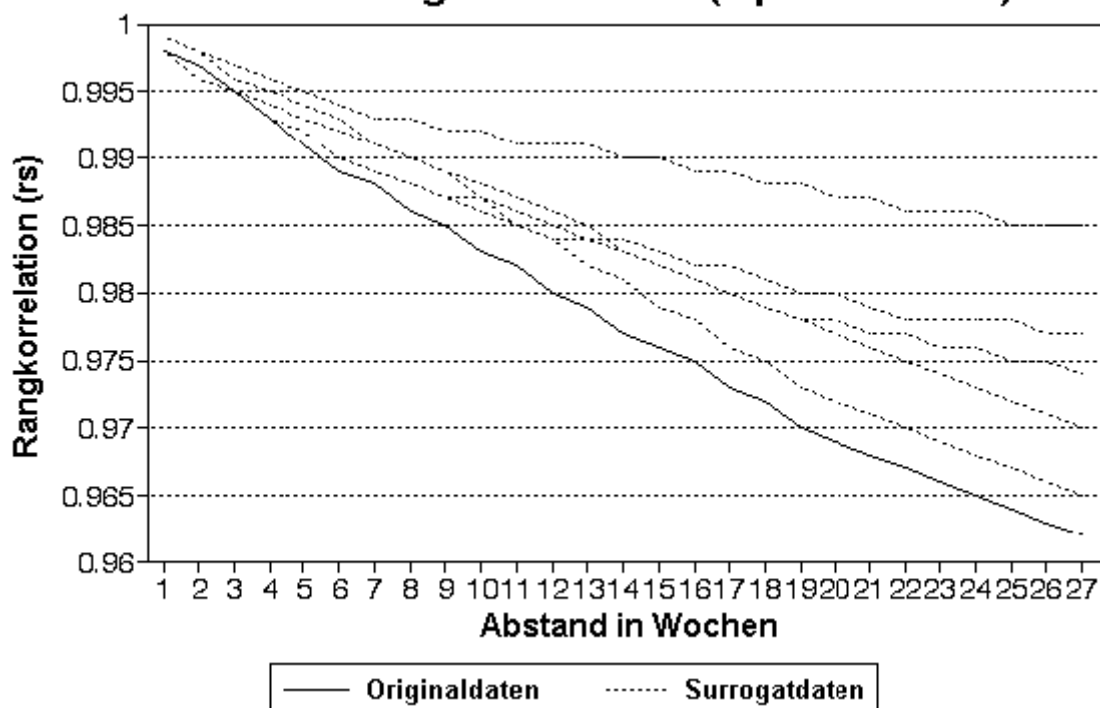
The trend is still contained in the data but it only shifts the whole curve upwards a bit. It looks rather random, doesn't it? Let us assume the moves were random. If this was true, we could mix the sequence of events in a (pseudo) random order, without influencing their predictability. Mean, standard deviation and range of the data would not be affected by exchanging the positions. This is how we produce the surrogate data. In the following figures we will show original data and we will oppose them to surrogate data in order to show, if the true DAX differs from the random DAXes and if it does, what the differences would be. Next you will see the true DAX (solid line) and five surrogate versions of the DAX (dotted lines) in logarithmic scale.

## Deutscher Aktienindex DAX 1980 - 2001



We can see that surrogate data can give rise to similarly looking curves. This is of course not a proof of the „random walk“ hypothesis. If we test for auto correlation of data, for example with Spearman's rank test, we do not measure merely possible linear but also monotonous association. The test can be applied to a broader range of problems than the better known product moment correlation test of Pearson, which assumes binormal data. Using Spearman's test we compare again the true DAX with the reconstructed surrogate indices.

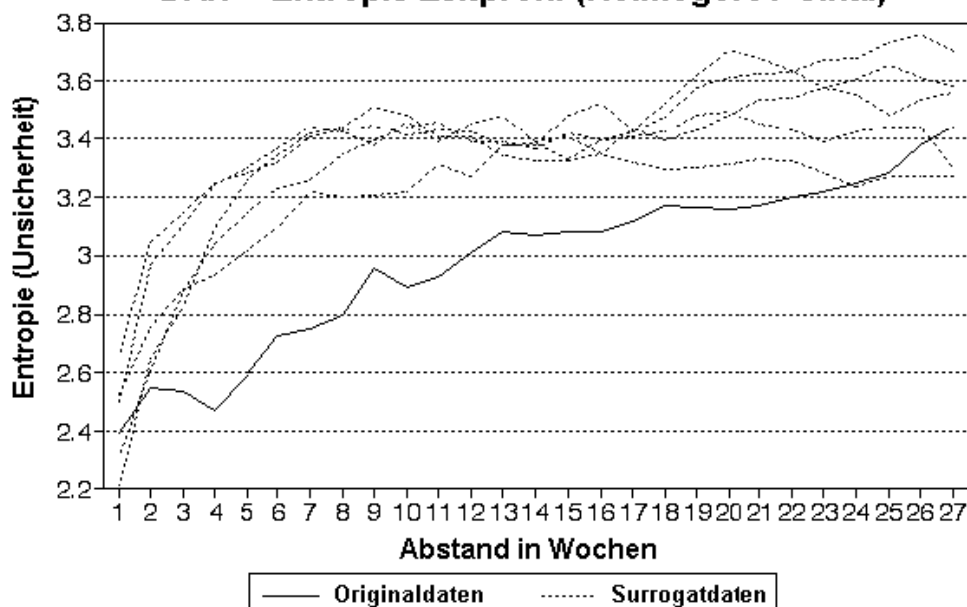
## DAX Rangkorrelation (Spearman rs)



The figure shows that for the first few weeks there is almost no difference between the DAX and its surrogate counterparts. In the long run the random sequences auto correlate even better than the original data. The rank correlation test will recognize also non linear associations between data, but only if these are monotonous. The following tests are not restricted in this way.

The opposite to information is called (information-) entropy. It is a measure of our uncertainty regarding the outcome of a certain event. Here the entropy graph shows how unsafe our predictions are on average if we want to forecast the DAX for a time span of one week up to a half year.

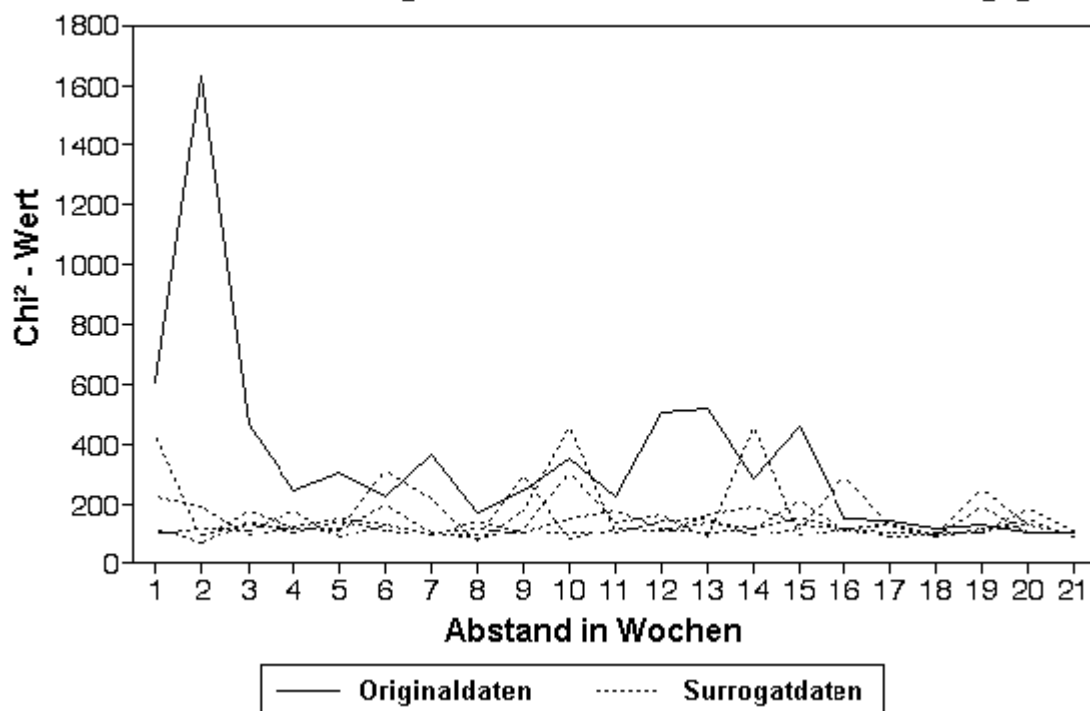
## DAX - Entropie Zeitprofil (Kolmogorov-Sinai)



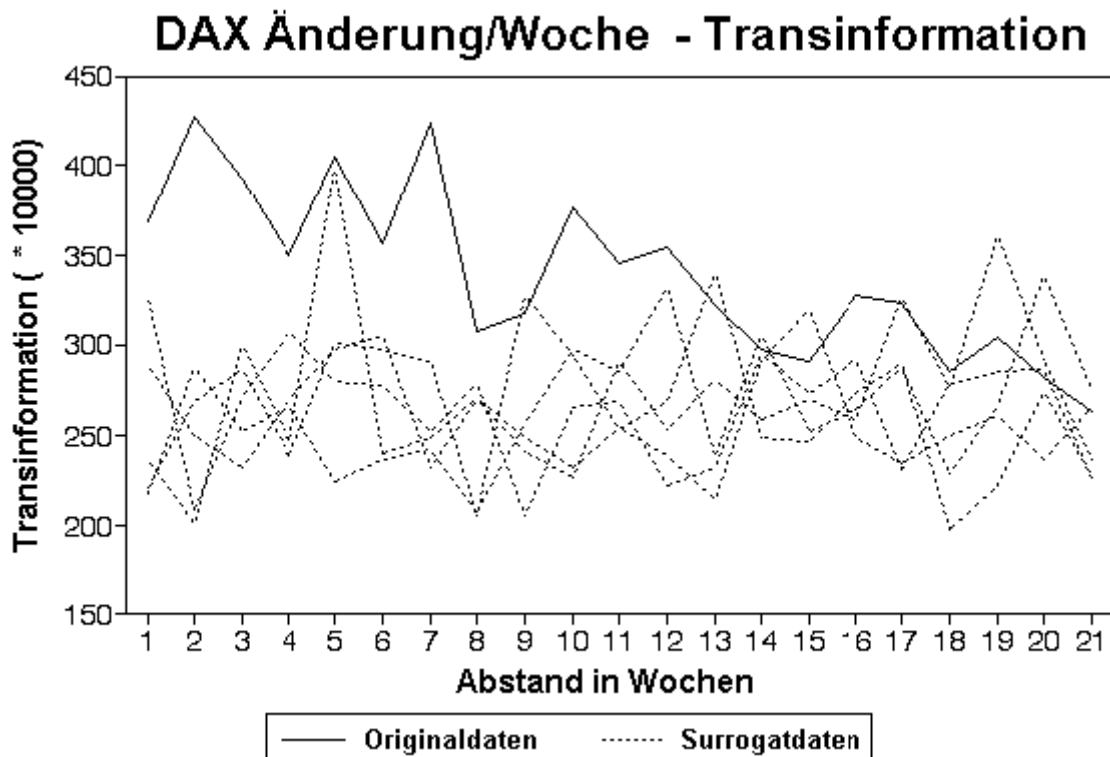
It becomes clear that our uncertainty increases with forecasting time. Apparently we cannot even make a forecast for six months which deserves the name, that is with better precision than we could forecast random events. But for a short time span our uncertainty is markedly less than it would be if the events occurred at random. And within this time span better forecasts are possible, at least theoretically.

In the information sciences the  $\chi^2$ -test of independence is of utmost importance. It measures any kind of association between data, may they be linear or non linear. The test is for example routinely applied to validate the quality of (pseudo-) random number generators. High  $\chi^2$  readings denote strong dependence. With this test the sequence of DAX returns was examined if there were associations among individual movements. Surrogate data should not have associations. This is a somewhat different comparison as opposed to the rank correlation and entropy profile, because in those tests we investigated the sums of returns over different time ranges.

### DAX Änderung/Woche - $\chi^2$ -Test auf Abhängigkeit



This test shows the biggest differences in the first three weeks and again a small window around twelve to thirteen weeks. Surrogate data do occasionally contain outliers, but these are isolated singular events which would be averaged out. After about sixteen weeks original data and surrogates are indistinguishable. This test again shows that forecasts that are better than random have short time limits. A further test which could help us decide if subsequent weekly moves in the DAX are random or obey to a hidden order, is the test of mutual information. Mutual information is in short the difference between unconditional and conditional entropy. The test shows how much information on an average is contained in the data. That means, how much our uncertainty would decrease if we had that additional information.



We can easily observe that the information on future moves of the DAX decreases rapidly with time. Mutual information of surrogates stay at a low level. Single outliers are isolated.

Well, what does all this tell us and what can we buy for it? One message is that long time forecasts are very unsafe. If we want to know how the index will be a half year from now, we cannot read it from the available data. We could give an almost equally valid forecast by throwing dice. And this is not only true for our own forecasts. It applies to the forecasts of all other experts and consultants as well. They cannot take more information out of the data than are contained in them. Additional information could be gained from daily data, but the time limits for reasonably accurate forecasts will not improve markedly.

But the comparison of true market data with surrogates leaves a second message for us. Market data do contain information about the likelihood of future moves. If the data contain information about the near future, this information could perhaps help us to identify a trend. The trend indicator should be robust, that is it should ignore small erratic moves, but it should not react too slowly to important changes. We have to look for acceptable compromises. The following figure shows that suitable compromises indeed exist.

Using the first half of the time series, that is from January 1980 to April 1990 a suitable trend finding formula was established, which would estimate a trend for the next week. Each week it would be decided, if a DAX share should be bought (held) or sold (not bought). Basis for the decision is the trend prediction. Shares are bought or held if an upward trend is predicted, shares are sold, or not bought respectively if a down trend is expected. This method was tested without modification of the parameters for the next eleven years and compared to 'buy and hold'.

In the figure you see the DAX in logarithmic scale. Starting with 5th May 1990 the original curve splits into three curves. The DAX curve is continued in black until August 24th, 2001. The grey and white curves show returns that would have been made with two hypothetical trading systems. The only difference between the grey and white curves is that white would move out of the market and ignore a positive trend forecast if it had experienced two successive losses. Both systems sold on 8th June and went out of the market which had a strong down trend then. The systems would not have suffered the severe DAX losses after 11th September.



Trading systems must make profit, but at the same time they have to reduce risk. It is quite easy to agree upon a profit measure, but with risk criteria it is more difficult. Some use the variance of returns as a risk measure, but variance treats wins and losses likewise as risk, which is not convincing. However, variance of returns is a common risk measure and therefore it is considered here. Other risk measures are the biggest individual loss and the biggest cumulative loss. The latter is found by summing up returns over various sliding time frames. Many successive losses eventually interspersed by some small gains may add up to very big losses if losses outweigh the gains over a considerable time. You will find some extreme values highlighted.



### German stock index DAX (May 1990 - Aug 2001) - Performance Table

Comparison	buy/hold	System I	System II
Gain	104.1 %	117.1 %	134.8 %
Variance	7.63 (% <sup>2</sup> )	3.65 (% <sup>2</sup> )	3.35 (% <sup>2</sup> )
Gain (% b/h)	100 %	109.6 %	129.5 %
Variance (% b/h)	100 %	47.6 %	42.6 %
Gain (Unit Variance)	100 %	230 %	304 %
Time invested	591 wks = 100%	353 wks = 59.5%	334 wks. = 56.5%
Transactions (Nos.)	2	100	110
Buy/Sell Cycles (Nos.)	1	50	55
Cycle Length (avg.)	591 wks.	7 wks.	6 wks
Winning weeks	328 = 55.5 %	201 = 56.9 %	193 = 57.8 %
Losing weeks	261 = 44.2 %	151 = 42.8 %	140 = 41.9 %
Winning week (avg.)	+2.09 %	+1.99 %	+2.00 %
Losing week (avg.)	-2.23 %	-1.87 %	-1.80 %
Biggest Single Loss	-14.1 %	-9.5 %	-8.4 %
Cumulated Loss 5 wks.	-23.1 %	-16.4 %	-14.7 %
Cumulated Loss 10 wks.	-42.2 %	-14.1 %	-11.8 %
Cumulated Loss 20 wks.	-33.4 %	-13.7 %	-12.1 %
Cumulated Loss 40 wks.	-28.8 %	-21.2 %	-16.2 %
Cumulated Loss 60 wks.	-30.7 %	-22.8 %	-17.8 %
Cumulated Loss 80 wks.	-37.7 %	-18.7 %	-13.7 %
Cumulated Loss 100 wks.	-9.9 %	-5.9 %	-5.7 %

From the table we can read for example that there was at least a single ten week period within which the DAX lost 42.2 % of its value. The other data on cumulative losses should be interpreted accordingly. Here the time frames were not continuously sampled. There may be other time frames than those given in the table, where even bigger losses appeared. For true trading systems this would have to be done more thoroughly.

The trading systems introduced here are far better than 'buy and hold' as regards profit and loss. However with the latter method there is only one cycle and thus only 2 transactions, while trading systems would have transaction costs more often than a hundred times. It can be argued that trading systems would be in the market only about 60% of time. The remaining time the money could have been invested in the money market, say on a deposit account. It would be no problem to consider costs for brokerage but these vary considerably therefore they were disregarded.

Even a good model may be wrong at times. This is only in part the consequence of the markets being open systems, which underlie influences from outside. More important is the fact that markets are not mathematical or physical systems which obey to mathematical logic and/or natural laws. Markets are social systems which interact through the exchange of information among their participants. Since human beings can only perceive and process a limited amount of information mistakes and errors are quite frequent. Even if errors might be recognized later on and wrong decisions could be corrected it is advisable to safeguard against unforeseeable events.

Just how advantageous it can be to reduce risk systematically becomes clear with method 'white', which is superior to the two other systems particularly at structural breaks, i.e. if the trend reverses abruptly. For estimating the trend only the limited information contained in the market data is used and forecasts are made for one week only. But since trends usually keep on for a while the engagements in the market or absence from the market continues for several weeks, on an average for six to seven weeks, depending on the trading system. Therefore transaction costs would not have to be paid every week.

The method is robust in several respects. It will work at times of different liquidity. It may use data series which do not contain only weekend closes, but can tolerate occasional mid week closes. No expensive equipment is needed and no expert has to be considered. And last not least the method worked without change over many years, and it still works, which is astonishing given that markets are adaptive dynamic systems (posses the ability to learn).